The Love of Numbers

Everyone knows, from an early age that ten x ten = 100. But, we can only write this result in that fashion because we are using a base ten number system.

Suppose we used a base 8 number system (I am sure we would have developed a base 8 number system IF we had 8 fingers on our two hands, we have a base ten system precisely because we have ten fingers on our two hands).

In a base 8 number system the digits used would be 0, 1, 2, 3, 4, 5, 6, 7, only

To write the number "eight" in base 8 we use "10", which means one of our base (8) and no units.

Thus the first 40 counting numbers in base eight would be:

1, 2, 3, 4, 5, 6, 7,1 0, 11, 12, 13,1 4, 15, 16, 17, 20, 21, 22, 23, 24, 25, 26, 2 27, 30, 31, 32, 33, 34, 35, 36, 37, 40, 41, 42, 43, 44, 45, 46, 47, 50

Therefore, in base eight we write ten x ten as 144 (64+32+4)

Other base systems have many uses, for instance all computers are programmed in binary code, ie numbers written in base two. You may think that base two would limit your options, but look at all the wonderful things that computers can do, by just utilising a code system where the only two digits are 0 or 1. Thus ten in base 2 is written as 1010. In base two: ten x ten = 110010

To write the whole sum in base two we write 1010 x1010 = 110010

Some of you may recall that I proved a very interesting result about Pythagoras' Theorem using numbers in bases 3, 4, and 5. (The result: if a, b, c, are all integers and $a^2 + b^2 = c^2$ then the product abc is exactly divisible by 60). This result is very difficult to prove just using base ten.

Advent of the Calculator

The discovery of the scientific calculator has been a wonderful boon to mathematics, it has taken all the hack work and drudgery out of long calculations and allowed the mathematician to spend more time on the logic of the problem. The down side to this has been an alarming decline in the ability of students to mentally calculate even the simplest problem, eg 71-69, or more practically, if a person went into a shop and purchased 3 items each valued at about $15, he/she should realise that they would receive about $5 change from $50.

Learning numbers can be fun. There are many easy ways to learn tables and simple number skills. Although these are necessary skills, there are many tricks and wonderful facts that can be used to give the novice genius a real "wow factor" to pique their interest and start them on the road to genius. All that is required is to apply some algebraic skills to arithmetic.

Consider the identity: $(a-x)(a-y) = a^2 - (x+y)a + xy$: Using this identity even the modest student can be shown how to mentally calculate the product of two numbers in the "90's".

Eg 97 x 94 = 9118. In this example $x = 3, \& \ y = 6, a = 100$ for all examples of this type

The first two digits are given by $100-(x+y)=91$; the last two digits are just the product $x\times y=18$ giving the result as above 97 x 94 = 9118. This technique can also to apply to numbers in the "80's" but the mental calculations become more onerous. Try a few and check your results.

In a previous newsletter it was explained how to square numbers ending in 5. I quote:

Squaring numbers that end in either "5" , or "0.5" or else $\frac{1}{2}$ is easy using the following trick.

Eg, to evaluate 65^2 (this means 65 x 65) you perform the following steps.

1. ***Multiple the "6" by the next number after the "6", ie perform 6 x7 = 42***
2. ***Place the two digits "25" after the 42, thus 65 x 65 = 4225***

The logic behind this result is a direct application of $(a+x)^2=a^2+2ax+x^2$ which can be re-factorised as $(a+x)^2=a(a+2x)+x^2$ In the above example $a=60, x=5, a+2x=70$ thus $a(a+2x)=60\times 70=4200$ giving the first two digits formed by 6 x 7 and the last two digits are obtained thus: $x^2=5^2=25$ This expansion can also be used to mentally square any number from 1 to 100 because for all such numbers n we can find an a such that the difference between a & n has a maximum absolute value of 2. Two examples are given below:-

67 x 67 = 65 x 65 + 4 x 65 + 4 = 4489 note: $67^2=(65+2)^2=65^2+4\times 65+4$

68 x 68 = 70 x 70 – 4 x 70 + 4 = 4624 note: $68^2=(70-2)^2=70^2-4\times 70+4$

This is more mentally challenging, even for the already recognised talented student (remember all students are gifted and talented, it is just a shame that not all of them recognise it just yet, but follow the guru and you will).

Succeeding in challenges like the above help the student realise his/her untapped potential. There are many other "short cuts" to mental calculations, many discovered by Jakow Trachtenberg in the early 1940"s, as an exercise you may research this genius on the web. One of his discoveries involves division by 91, and this is an easy process for students to acquire.

Eg $\frac{71}{91}=0\cdot 7802197$ *or* $\frac{48}{91}=0\cdot 5274725$ The process is deceptively easy, let me explain for both cases.

Case 1.

1. The first digit after the decimal point is the first digit of the top of the fraction (ie the numerator), here = 7
2. The second digit is the sum of the two digits of the numerator, 7 + 1 = 8
3. Third digit is the difference between the last digit on the numerator and 1 - 1 = 0
4. The next three digits are the difference between 9 and the first three digits. Thus 9 – 7 = 2 and 9 – 8 = 1, 9 – 0 = 9, giving the 219 part of the answer

5. The last digit is the same as the first digit, the answer is NOT correct to seven decimal places because it does not take into consideration that the eighth digit may be greater than or equal to 5

Case 2.

1. The first digit after the decimal point is the first digit of the top of the fraction (ie the numerator), here = 4
2. The second digit is the sum of the two digits of the numerator, 4 + 8 = 12. Note, as with the case of all addition we have to carry the ten, so the second digit stays 2, but the first digit now becomes 5, making the first two digits 52
3. Third digit is the difference between the last digit on the numerator and 8 - 1 = 7
4. The next three digits are the difference between 9 and the first three digits. Thus 9 – 5 = 4 and 9 – 2 = 7, 9 – 7 = 2, giving the 472 part of the answer
5. The last digit is the same as the first digit, the answer is NOT correct to seven decimal places because it does not take into consideration that the eighth digit is greater than or equal to 5

There are many other wonderful examples out there for the student who has a love of numbers. Check our website and look for the Old Master Coach on Youtube, he is here to make mathematics fun for everyone.